Figure 1: Range $\tau$:
$R_{R_\tau}$ = radiation range of $\tau$ (at $\rho_r$ density)
$R_\mu$ = radiation range of $\mu$ (at $\rho_r$ density)
$R_{W_\tau}$ = electroweak range of $\tau$ (at $\rho_r$ density)
$R_{\tau_o}$ = boosted $\tau$ lifetime range

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# The role of $\nu_\tau$ ultrahigh energy astrophysics in km³ detectors

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### Abstract

We show that the expected $\nu_\tau$ signals, by their secondary tau tracks, in $Km^3$ detectors at highest cosmic ray energy window $1.7 \cdot 10^{21}\,\text{eV} > \text{E}_\tau > 1.6 \cdot 10^{17}\,\text{eV}$, must overcome the corresponding $\nu_\mu$ (or muonic) ones. Indeed, the Lorentz-boosted tau range length grows (linearly) above muon range, for $E_\tau \gtrsim 1.6 \cdot 10^8$ GeV and reaches its maxima extension, $R_{\tau_{\max}} \simeq 191$ km, at energy $E_\tau \simeq 3.8 \cdot 10^9$ GeV. At this peak the tau range is nearly 20 times the corresponding muon range (at the same energy) implying a similar ratio in $\nu_\tau$ over $\nu_\mu$ detectability. This dominance, however may lead (at present most abundant $\nu_\tau$ model fluxes) to just a rare spectacular event a year (if flavor mixing occurs). Lower energetic $\tau$ and $\nu_\tau$ signals ($\bar{\nu}_e e \rightarrow \bar{\nu}_\tau \tau$, $\nu_\tau N \rightarrow \cdots$) at energy range ($10^5 \div 10^7$ GeV) may be more easily observed in km³ detectors at a rate of a few ($\bar{\nu}_e e \rightarrow \bar{\nu}_\tau \tau$) to tens event ($\nu_\tau N \rightarrow \tau+$ anything) a year.

## 1. Introduction

High energy astrophysics is waiting for the new neutrino telescope generations able to reveal the expected TeV (and above) energetic neutrinos ejected by active nuclei (AGN) blazars [1] as well as from galactic supernova relics or galactic mini-blazars. The common theoretical argument in favor for neutrino cosmic ray (c.r.) source is the last experimental evidence of extragalactic TeV photon sources (Markarian 421,501) and the unique neutrino transparency through cosmic 2.75 K° B.R. from cosmic distances. Secondary atmospheric neutrinos will play a negligible role at high ($\gtrsim 10^4 \div 10^5$ GeV) c.r. energy. The common expected neutrinos are of electronic ($\nu_e$, $\bar{\nu}_e$) and muonic ($\nu_\mu$, $\bar{\nu}_\mu$) nature because of the "low energetic" pion progenitor masses ($m_{\pi^\pm}$), and their consequent easier and larger productions in proton-proton scattering. However, at very high energy ($E_p > 10^{15}$ eV)

the p-p scattering may lead, by charm production, to other secondary charmed hadrons able to decay also in tau leptons; this possibility allows (at least above $10^{15}$ eV) the production of $\nu_\tau$ component as abundant as $\nu_e$, $\nu_\mu$ ones. Moreover, flavor mixing and oscillation like ($\nu_\mu \leftrightarrow \nu_\tau$), even at most wide and unexplored parameter ranges ($1 > \sin^2 2\theta_{\mu\tau} > 0$, $\Delta m^2_{\mu\,\tau} \ll 0.2\,\mathrm{eV}^2$) are well compatible with characteristic large galactic and huge cosmological lengths $L_g \sim 10^{24}$ cm, $L_c \sim \frac{c}{H_o} \sim 10^{28}$ cm. Indeed the flavor oscillation length is

$$L_\nu(\nu_i \to \nu_j) = 1.23 \cdot 10^{16} \text{ cm } \left(\frac{E_\nu}{10^{20}\,\mathrm{eV}}\right)\left(\frac{\Delta m_{i,j}}{\mathrm{eV}}\right)^{-2} \ll L_g,\, L_c\,. \tag{1}$$

Therefore flavor mixing may easily lead to an abundant $\nu_\tau$ production. However, ultrahigh energy $\nu$ interactions with matter, deeply overviewed and summarized by last R. Gandhi, C. Quigg, M. H. Reno, I. Sarcevic reports [2] received little attention to the $\nu_\tau$ role (probably because of the very short unstable lifetime of the $\tau$ lepton: ($\tau_\tau \sim \left(\frac{m_\tau}{m_\mu}\right)^5 \tau_\mu \sim 3 \cdot 10^{-13}$ sec)). Nevertheless, a first important UHE $\nu_\tau$ role at PeV energies has been noted also recently by J. Learned and S. Pakvasa [3]; in particular, these authors stressed that a characteristic hadronic behaviour at the initial event of the $\nu_\tau$ interaction and at the end shower of the lepton $\tau$ track: a "double bang" signal.

Here we underline the dominant and key role of $\nu_\tau$, $\bar\nu_\tau$ tracks signatures by their secondary tau at much higher energies ($E_\nu \gtrsim 10^{17} \div 10^{20}$ )eV over muon ones because of the large Lorentz factors and the consequent longer tau tracks. This relativistic tau "longevity" makes the heaviest lepton the most easily detectable above $5 \cdot 10^{17}$ eV (or $10^{17}$ eV in the rock) in a km$^3$ detector. Lower energetic $\nu_\tau(10^7\,\mathrm{GeV} > E_\nu > 10^5\,\mathrm{GeV})$ may be more easily observed because of a more aboundant primary flux as discussed in the conclusion. Their discovery may lead to the first "direct" evidence for the $\nu_\tau$ existence, may open the most fashionating window at the highest c.r. astrophysics frontiers and it may prove the deepest secrets of most powerful cosmic accelerators.

## 2.  The tau radiation length versus the muon one

Muons are commonly known as the most penetrating charged cosmic ray because their radiation length is much longer (at high energy) than the corresponding electron one. Indeed, the muon radiation length at high energy is roughly $\left(\frac{m_\mu}{m_e}\right)^2$ longer than that of the electron, because (see Jackson [4], eq. 15.48) the characteristic leptonic bremsstrahlung radiation length $b_L^{-1}$ is found classically:

$$b_L^{-1} = \left[\frac{16}{3} Z^2 N \left(\frac{c^2}{\hbar c}\right)\left(\frac{c^2}{m_L c^2}\right)^2 \ln\left(\frac{\lambda 192 m_L}{Z^{1/3} m_e}\right)\right]^{-1}, \tag{2}$$

where $N$ is the atomic number density which is proportional to the Avogadro number times the average density, $\lambda$ is an a-dimensional factor near unity, $m_L$ is the lepton $(e, \mu, \tau)$ mass and $Z$ is the target nuclear charge. Therefore neglecting the "slow" logarithmic mass dependence, the radiation length $b_L^{-1}$ is mainly proportional to the square of the lepton mass $m_L$. The radiation loss by pair production would be, at higher energies, the ruling one (over bremsstrahlung and over the negligible photo nuclear losses). Nevertheless, all the radiation lengths grow in similar form i.e., as the square of the lepton mass $(\sim m_L^2)$. The reason of it is in the probability amplitude of the corresponding Feymann diagram, where an exchange of a virtual photon by a nuclei and by the incoming relativistic lepton leads to the emission of a high energy photon, or an electron pair. The process amplitude is roughly proportional (because of the lepton mass presence in the propagator) to the inverse of the lepton mass $(m_L)$. The consequent cross section and its inverse (roughly proportional to the radiation length) decrease (or grow) consequently as $\left(\frac{m_L}{m_c}\right)^{-2}$ (or $\left(\frac{m_L}{m_c}\right)^2$) as it has been found classically and experimentally in Eq. (2). Therefore the most penetrating lepton must be the heaviest ones, i.e. the tau leptons. On the other hand, the lifetime of the unstable tau lepton, being proportional to the inverse of the fifth power of its mass $\tau_c \simeq \left(\frac{m_\mu}{m_c}\right)^5 \tau_\mu$, makes its track extremely short: $c\tau_\tau = \gamma_\tau 9 \cdot 10^{-3}$ cm (with respect to the muon ones). At highest energies $(E_\tau \gg 100\,\text{TeV})$ the huge Lorentz factor boost the observed short tau lifetime and increase its value linearly with energy while the corresponding muon tracks already reached, in the water or in the rock, a nearly steady maxima (a logarithmic growth) of a few kilometers length. Consequently, at highest energy $(E_\tau \gtrsim 5.6 \cdot 10^8\,\text{GeV}$ in water, $1.6 \cdot 10^8\,\text{GeV}$ in the rock) the tau radiation length will be the longest one and the cosmic tau neutrino rays, $\nu_\tau$, $\bar{\nu}_\tau$ (if abundant as other flavors) will be the dominant source of signals in km$^3$ detectors over other leptons at the same energies. Finally, as for the muons, also the tau radiation length will reach a maxima extension at the highest energies $(\sim 4 \cdot 10^9\,\text{GeV})$ for two main energy losses:

a) The electromagnetic radiation losses (pair production).

b) The electroweak interactions and losses with matter (mainly nucleons).

The latter processes is the main restrictive constraint on tau tracks (in water and rock) at $E_\tau \gtrsim 5 \cdot 10^9$ GeV and it provides a maxima radiation length comparable to those of the neutrino at same energies $(\gtrsim 200\text{km}$ in the rock, $\gtrsim 420\,\text{km}$ in water) which will be discussed further in detail, below. The growth of the lepton $\tau$ radiation length and its (proportional) detectability leads to a fundamental and dominant role of $\nu_\tau$ UHE $(\gtrsim 10^8\,\text{GeV})$ astrophysics, in a near future km$^3$ or larger neutrino telescope. Contrary to present arguments, we remind that the *absence* of $\nu_\tau$ fluxes when flavour obscillation are forbidden, at lower energies $(10^{11} \div 10^{13}\,\text{eV})$ has already been considered by us [5]

(in absence of flavor oscillations), in order to bound the properties of any hypothetical heavy fourth neutrino generation clustered, as cold dark matter, in galactic halos.

The large ratio of the $\tau$ radiation length over those of the muons, reaching in principle a maximal factor $\sim \left(\frac{m_\tau}{m_\nu}\right)^2$ (or at least two order of magnitude), might imply a corresponding ratio in the detectability of the two leptons at those energies ($10^8 \div 10^{10}$ GeV); however nuclear interactions as shown in more detail in the text make this ratio smaller ($\sim 20$).

Finally, the secondary muons "tail" $\mu$ due to $\tau$ decays ($\mu_\tau$) will also increase by a large fraction ($\sim 100\%$) the indirect $\tau$ (and $\nu_\tau$) detectability. Moreover, the most probable $\tau$ ($\gtrsim 60\%$) hadronic decay (and its consequent shower) or its electroweak nuclear shower will lead as it has been noted [3] to an unambiguous "hadronic" jet signature in underground detectors, contrary to common "quite" one-track muon leptonic decays.

## 3. The source of high energy tau neutrinos

As we already noted in the introduction at very high energy ($E_p > 10^{15}$ eV) $p$-$p$ scattering may lead, by charmed hadronic production, also to a secondary tau (and a neutrino tau $\nu_\tau$), whose abundance may be as proliferous as other flavor ones ($\nu_e$, $\nu_\mu$). In most models these neutrinos are expected from Active Galactic Nuclei (AGN) or blazars [1].

Ultra-high energy neutrinos $\nu_e$, $\nu_\mu(\bar{\nu}_e, \bar{\nu}_\mu)$ ($E_\nu > 10^{19}$ eV) may also be born copiously by photopion production of high energy proton (and neutron) ($E_p \gg 10^{19}$ eV) onto cosmic 2.75K° BBR and galactic radio waves background. Unfortunately, these abundant photopion productions at $10^8 \div 10^{12}$ GeV cannot in general produce direct tau neutrinos. Nevertheless, there are other related expected able to lead also in this interesting energy range ($3 \cdot 10^8$ GeV $\div 10^{12}$ GeV) to primary or secondary high energy tau and $\nu_\tau$:

a) Hadronic (charm or beauty) showers due to downward or horizontal high energy neutrinos ($\nu_e$, $\nu_\mu$) interaction (mainly nuclear) in the Earth by charged neutrino-electron interactions; ($\bar{\nu}_e e \to \bar{\nu}_\tau \tau$) at the resonance $W^-$ mass peak are relevant only at energy peak $E_\nu \sim 6 \cdot 10^6$ GeV to be discussed at the end.

b) Flavor oscillations $\nu_\mu \to \nu_\tau$ (as well as $\nu_e \to \nu_\tau$) at the widest and even unexplored parameter ranges: ($1 > \sin^2 2\theta_{e\tau} > 0$), ($1 > \sin^2 2\theta_{\mu\tau} > 0$); $\Delta m_{i,\tau}^2 \ll 0.2\, \text{eV}^2$ [6]. For any realistic neutrino mass these parameter rays may be satisfied.

Indeed, flavor oscillations lengths, as already mentioned, even stretched by the huge Lorentz factor is in general below to characteristic cosmological $\frac{c}{H_o}$ distances: (see Eq. 1).

Finally we remind that ultrahigh energy tau pairs production, by high energy photon ($E_\gamma \gtrsim 5 \cdot 10^{21}$ eV)-photon (B.B.R. at 2.7 K) Compton Scattering, may also take place, but at a very low rate.

Therefore we shall consider in the following the neutrino and anti neutrino tau cosmic ray flows as abundant (or comparable) as all the other flavors ones; in conclusion even in absence of any flavor mixing the $\nu_\tau$ secondary (or $\tau$) must exist by $\nu_e$, $\nu_\mu$ hadronic secondaries more probably along an horizontal plane ring (where the $\nu_\tau N$, $\tau N$ interactions lengths are comparable to the detector depths). Astrophysical sources and fluxes for such a high energy ($\gtrsim 10^7 \div 10^{12}$ GeV) neutrinos have been modeled by many; we refer mainly to the flux calculated by Stecker and Salamon [7] which will probably dominate in the energy range ($10^7 \div 3 \cdot 10^8$ GeV), labeled by AGN-SS (in ref[2], in Fig. 18) due to $p$-$p$ scattering at source; in this range we *must* expect primary $\nu_\tau$. We also refer to the photopion production of cosmic rays and the secondary neutrino flux ($\nu_e$, $\nu_\mu$, $\bar\nu_e$, $\bar\nu_\mu$) considered by Yoshida and Teshima [8] either for turn-on time at maximal redshift $z = 2$ (labeled by CR–2 in [2]) and redshift $z = 4$ (labeled by CR–4 in [2]). For the last two models the expected neutrino maxima fluxes at the neutrino energy range $3 \div 5 \cdot 10^9$ GeV reaches a value of $\sim 10^{-18}$ cm$^{-2}$s$^{-1}$sr $\div 10^{-17}$ cm$^{-2}$s$^{-1}$sr$^{-1}$ i.e. fluxes comparable to those observed at the same energies in known cosmic rays on Earth.

## 4. Ranges of ultrahigh energy tau lepton

As we already mentioned the radiation length $b_\tau^{-1}$ for tau lepton, due mainly to pair production in Eq. (2), will increase the range of tau tracks (energy dependent) with respect to corresponding of muons, as soon as the Lorentz boost ($\gamma_\tau$) will reach large values ($\gamma_\tau \gtrsim 10^8$) and as long as the electro weak interaction with nucleons will not bind their growth.

The radiation length $b_\tau^{-1}$ will play a role in defining the tau range by the general energy loss equation:

$$-\frac{dE_\tau}{dx} = a(E_\tau) + b_\tau(E_\tau)E_\tau \ , \tag{3}$$

where $a$ and $b$ are slow energy variable functions respectively for ionization and radiation losses. The asymptotic radiation length $b_\tau^{-1}$ at high energies $E_\tau \gg 10^{15}$ eV is related to the corresponding muon one by this approximated relation derived by classical bremsstrahlung formula in Eq. (2) scaled for the two different lepton masses:

$$b_\tau \simeq \left(\frac{m_\mu}{m_\tau}\right)^2 \cdot \left[\frac{\ln\left(\frac{\lambda 192\, m_\tau}{z^{1/3} m_e}\right)}{\ln\left(\frac{\lambda 192\, m_\mu}{z^{1/3} m_e}\right)}\right] \cdot b_\mu \cong \frac{b_\mu}{219} = 1.78 \cdot 10^{-8}\, \text{cm}^{-1} \rho_\text{r}^{-1} \ , \tag{4}$$

where, $\rho_r^{-1}$ stand for relative adimensional density in water unity, and where in the present energy range, $E_\tau \gg 10^5 \, \text{GeV}$, we assumed that the experimental phenomenological coefficient as in Ref. [2]: $b_\mu \simeq 3.9 \cdot 10^{-6} \, \text{cm}^{-1} \rho_r^{-1}$. The corresponding radiation length $b_\tau^{-1}$ is: $b_\tau^{-1} \simeq \dfrac{561 \, \text{km}}{\rho_r}$ , The ionization coefficients values are: $a_\tau \simeq a_\mu = 2 \cdot 10^{-3} \text{GeVcm} \rho_r^{-1}$. The integral of the energy loss equation will lead, from the radiation length $b_\tau^{-1}$, to a larger, energy dependent, radiation range $R_{R_\tau}$:

$$R_{R_\tau} \equiv \frac{b_\tau^{-1}}{\rho_r} \, \ln \, \frac{a_\tau + b_\tau E_\tau}{a_\tau + b_\tau E_\tau^{\min}} \simeq \frac{b_\tau^{-1}}{\rho_r} \, \ln \, \frac{E_\tau}{E_\tau^{\min}} \, . \tag{5}$$

The last approximation occurs because of the smallness (for $E_\tau \gg 10^5 \, \text{GeV}$) of the ionization factor $a_\tau$ with respect to $b_\tau E_\tau^{\min}$ and $b_\tau E_\tau$ terms.

In the Earth, according to the preliminary Earth Model [1] on the first few km the relative density $\rho_r$ is unity in the sea, near 3 in the early depth rocks, around 5 in the first 1000 km Earth depths. Therefore the consequent tau radiation length from Eqs. (4)–(5) (for $\rho_r \sim 5$), $E_\tau \gg 10^4 \text{GeV}$ becomes:

$$R_{R_\tau} \cong 1292 \, \text{km} \, \left( \frac{\rho_r}{5} \right)^{-1} \left\{ \frac{\left[ \ln \left( \dfrac{E_\tau}{10^8 \, \text{GeV}} \right) \left( \dfrac{E_\tau^{\min}}{10^4 \, \text{GeV}} \right)^{-1} \right]}{(\ln \, 10^4)} \right\} \, . \tag{6}$$

This extreme propagation range, comparable even to the Earth radius, is to be combined with and bounded by, the more restrictive tau lengths due to short tau lifetime, as well as by the range due to electro weak tau-nucleons interactions at the highest energies $(E_\tau \gtrsim 10^9 \, \text{GeV})$.

The role of tau lifetime and its free path length $R_{\tau_o}$, boosted by large Lorentz factors $\gamma_\tau = \frac{E_\tau}{m_\tau c^2}$, grows linearly with energies:

$$R_{\tau_o} = c \tau_\tau \gamma_\tau = 5 \, \text{km} \, \left( \frac{E_\tau}{10^8 \, \text{GeV}} \right) \, . \tag{7}$$

The electroweak tau-nucleon interaction range, $R_{W\tau}$, on the other side, decreases with tau energies in analogy with the corresponding ones for neutrino-nucleon scattering. In a first approximation the cross sections $\sigma(\nu_\tau N)$, at energy of interest $10^6 \, \text{GeV} \leq E_{\nu_\tau} \leq 10^{12} \, \text{GeV}$ may be described by a simple power law form, either for charged and neutral currents; because of the crossing symmetry

in the Feynman diagrams we may also write (following [2]) similar expressions for $\sigma(\tau N)$:

$$
\begin{aligned}
\sigma_{cc}(\tau N) &\simeq \sigma_{cc}(\nu_\tau N) = 4.44 \cdot 10^{-33} \text{ cm}^2 \left( \frac{E_\tau}{10^8 \text{ GeV}} \right)^{0.402} \\
\sigma_{Nc}(\tau N) &\simeq \sigma_{Nc}(\nu_\tau N) = 1.95 \cdot 10^{-33} \text{ cm}^2 \left( \frac{E_\tau}{10^8 \text{ GeV}} \right)^{0.408} \\
\sigma_{cc}(\bar\tau N) &\simeq \sigma_{cc}(\bar\nu_\tau N) = 4.3 \cdot 10^{-33} \text{ cm}^2 \left( \frac{E_\tau}{10^8 \text{ GeV}} \right)^{0.404} \\
\sigma_{Nc}(\bar\tau N) &\simeq \sigma_{Nc}(\bar\nu_\tau N) = 1.87 \cdot 10^{-33} \text{ cm}^2 \left( \frac{E_\tau}{10^8 \text{ GeV}} \right)^{0.41}
\end{aligned}
\tag{8}
$$

The corresponding averaged electroweak range $R_{W\tau}$ in the energy range of interest in water for a total (charged + neutral) cross sections $\sigma(\tau N) \simeq \sigma(\bar\tau N) \simeq 6.5 \cdot 10^{-33} \text{ cm}^2 \left( \frac{E_\tau}{10^8 \text{ GeV}} \right)^{0.404}$ is:

$$
R_{W\tau} = \frac{1}{\sigma N_A \rho_r} \simeq \frac{2.5 \cdot 10^3 \text{ km}}{\rho_r} \left( \frac{E_\tau}{10^8 \text{ GeV}} \right)^{-.404}.
\tag{9}
$$

The total tau range, $R_\tau$, is just the minimal value of the three above ones: the radiation one $R_{R_\tau}$ in Eq. (5), the lifetime one $R_{\tau_o}$ in Eq. (7), the present electroweak-nuclear one $R_{W\tau}$ in Eq. (9):

$$
R_\tau = \left( \frac{1}{R_{R_\tau}} + \frac{1}{R_{\tau_o}} + \frac{1}{R_{W\tau}} \right)^{-1}.
\tag{10}
$$

Let us notice that in the estimate of the electroweak range $R_{W\tau}$ we neglected the (otherwise) interesting electron-tau electroweak interactions in the atoms for two main reasons:

a) The tau-electron electroweak cross sections ($\tau e \to \tau e$) do not experience the (corresponding) resonant peak (as for neutrino-electron scattering: $\nu_\tau e \to W^- \to \tau \nu_e$) at energies $E_\nu \sim 6.10^{15}$ eV. The analogous resonant reaction ($\tau^+ e \to Z_o \to \nu_\tau \nu_e$) is forbidden by flavor conservation number. Tau and electrons may only interact weakly by electroweak exchange of a neutral virtual boson $Z_o$ and a photon.

b) Even in the above (not allowed) case of a resonant cross section at energy $E_\nu \sim 6 \cdot 10^{15}$ eV and cross section $\sigma_{\tau e} \sim 10^{-31}$ cm$^2$, the shortest tau lifetime and its range $R_{\tau_o}$, in Eq. (10), will mask and hide the short range $R_{W\tau}$ (due to hypothetical $e\tau$ "resonant" scattering).

It is important to consider the tau range $R_\tau$ at its characteristic regimes: when its value will overcome the corresponding muon one $R_\mu$ ($R_\mu = R_\tau$), when it will reach its maximal extension $R_\tau = R_{\tau \max}$, when it will be confined (because of nuclear interactions) at the highest energies to the same ranges as muon tracks ($R_\tau = R_{W\tau} = R_\mu$).

## 5. The critical energies for $\nu_\tau$ dominance

Let us define the first critical energy, $E_{\tau_1}$, where the tau range equals the muon one: $R_\tau = R_\mu$ from (Eq. (10)), (Eq. (5)) and (Eq. (2)) by substitution of $b_\tau$ with $b_\mu$. This equation may be easily solved noticing that at this energy range ($E_\tau \sim 10^8$ GeV) the shortest and main tau range is the lifetime one, $R_\tau \simeq R_{\tau_o}$; therefore the equation $R_\tau = R_\mu$ can be written as follows:

$$R_\tau \simeq R_{\tau_o} = c\tau_\tau \left( \frac{E_\tau}{m_\tau c^2} \right) = \frac{1}{b_\mu \rho_r} \ln \left( \frac{a + b_\mu E_\mu}{a + b_\mu E_\mu^{\min}} \right) ; \tag{11}$$

where one imposes $E_\mu = E_\tau$; numerically one finds that

$$5 \,\text{km} \left( \frac{E_\tau}{10^8 \,\text{GeV}} \right) \cong \frac{2.56}{\rho_r} \ln \left( \frac{E_\mu}{E_\mu^{\min}} \right) \text{km} . \tag{12}$$

defines the critical energy $E_{\tau_1}$ where tau track exceedes the muonic one. For water $\rho_r = 1$, and rock (in these depths $\langle \rho_r \rangle = 3$) the critical energy $E_{\tau_1}$ and the tau range $R_{\tau_1}$ are:

$$
\begin{array}{ccc}
E_{\tau_1} = 5.6 \cdot 10^8 \,\text{GeV} & E_{\tau_1} \cong 1.65 \cdot 10^8 \,\text{GeV} & \\
& \text{(water)}; & \text{(rock)} \quad . \qquad (13) \\
R_{\tau_1} = 28 \,\text{km} & R_{\tau_1} \cong 8.2 \,\text{km} &
\end{array}
$$

Here we considered $E_\mu^{\min} = 10^4$ GeV as in [1]. Let us remind that the analytical curve we are using in Eq. 5 for muons is a bit overestimated with respect to a more detailed study (Lipari, Stanev [9]) and therefore the present critical "analytical" value $E_{\tau_1}$ and the range $R_{\tau_1}$, might be larger than the real one (by a factor $1.5 \div 2$). Therefore from energies $E_\tau > 10^8$ GeV above the tau signal will overcome the muon ones. Moreover, the prompt secondary muons from tau decays or from tau hadronic pions decays (let us label them $\mu_\tau$), may in principle "double" the expected muonic fluxes; finally the characteristic tau hadronic decay ("bang" [3]) may leave a unique signal.

The linear growth of the tau range $R_\tau$, *in absence* of the $\tau N$ interactions, would reach a maximal radiation range $R_{T_\tau}$ (due to maximal $b_\tau^{-1}$ in Eq. (4)) as in Eq. (6), at least two orders of magnitude larger than the corresponding muon range ($R_\mu$). Indeed, it is possible to show that in such an *ideal* (no-electroweak interactions) case the relation $R_{\tau_o} = R_{\tau_R}$ would define an extreme energy $E_\tau \simeq 4 \cdot 10^{10}$ GeV and a corresponding range $R_{\tau_o} \simeq 2000$ km, much longer than the $R_{\mu R}$ range (at the same energy in the rock): $R_\mu(4 \cdot 10^{10} \,\text{GeV}) \cong 14 \,\text{km}$.

However, the *real* maximal tau range is bounded by the the more restrictive electroweak cross sections (in Eq. (8)) (as for neutrinos) and its range $R_{W\tau}$ (in Eq. (9)). The maximal tau range $R_{\tau_{\max}}$ is then defined by equal conditions in Eqs. (7) and (9), ($R_{\tau_o} = R_{W\tau}$):

$$R_{\tau_o} = 5 \,\text{km} \left( \frac{E_\tau}{10^8 \,\text{GeV}} \right) = \frac{2.5 \cdot 10^3 \,\text{km}}{\rho_r} \left( \frac{E_\tau}{10^8 \,\text{GeV}} \right)^{-0.404} , \tag{14}$$

whose solution is (for $\rho_r = 3$ as in the few hundred terrestrial km depths):

$$R_{\tau_{\max}} = 191 \, \text{km} \left(\frac{\rho_r}{3}\right)^{\frac{1}{1.404}} \; ; \; E_{\tau_{\max}} = 3.8 \cdot 10^9 \, \text{GeV} \left(\frac{\rho_r}{3}\right)^{\frac{1}{1.404}} \; . \tag{15}$$

For the peculiar case ("horizontal" neutrinos arrivals), in the sea, where we may assume $\rho_r = 1$, at a detector depth $\sim 10 \, \text{km}$, and a sea $\sim 20 \, \text{km}$ depth, one gets $R_{\tau_{\max}} \cong 418 \, \text{km}$, $E_{\tau_{\max}} \cong 8.36 \cdot 10^9 \, \text{GeV}$.

It is clear that from Eq. (15) the total maximal tau range $R_{\tau_{\max}}$ extends 20 times the corresponding muon range at the same energies.

Finally at higer energies also the electroweak interaction will bound the muon radiation range and it will make comparable both the taus and muons ranges. This will occur once the relation $R_\tau \simeq R_{W\tau} = R_\mu$ at the same energy $E_\mu = E_\tau$ (in Eqs. (2)–(5) and Eq. (9)) is satisfied; i.e. when

$$R_\mu \cong \frac{b_\mu^{-1}}{\rho_r} \ln \frac{E_\mu}{E_\mu^{\min}} = R_{W\tau} = \frac{1}{\sigma_W N_A \rho_r} \; . \tag{16}$$

This relation implies, for an adimensional density $\rho_r = 3$ a critical energy and ranges:

$$E_{\tau_2} \simeq 1.7 \cdot 10^{12} \, \text{GeV} \; ; \; R_{\tau_2} \simeq R_{W\tau} = R_\mu = 16 \, \text{km} \; . \tag{17}$$

Therefore from energy $E_{\tau_1} = 1.6 \cdot 10^8 \, \text{GeV}$ up to the energy $E_{\tau_2} \simeq 1.7 \cdot 10^{12} \, \text{GeV}$, the tau tracks will overcome the muon lengths and it will imply a dominant role for tau neutrino astrophysics (assuming, of course, a correponding $\nu_e$, $\nu_\mu$ spectra). Will this dominance be detectable in km$^3$ detectors? The answer, of course, depends on the unknown primordial cosmic flux: assuming, as in [2] a model flux labeled CR–2 and CR–4, whose event rates are summarized in Table 6 [2], the consequent tau event rate may be promptely derived by scaling the $R_\tau$ range in place of $R_\mu$ for effective km$^3$ volume made by the effective area A: $A\langle R \rangle$. The most optimistic rates for downward neutrinos ($D$-parton distribution, a CR–4 model [2]) above $10^7$ GeV may reach a muon event rate a year of $\sim 4.8 \cdot 10^{-2}$ and a corresponding tau rate just near the unity for the longest tau tracks at its maximal range extension $R_{\tau_{\max}}$ in Eq. (15). Therefore the $10^8 \div 10^{12}$ GeV energy window dominance of $\nu_\tau$ is at present models just at the edge of detectability at near future km$^3$ detectors. However, more yet unobserved and abundant ultrahigh neutrinos fluxes, may increase drastically our predictions. The $\nu_\tau$ and tau presence in the km$^3$ detectors may also be discovered by other indirect effects: for instance the presence of secondary relic muon bundles; indeed the hadronic jets due to $\nu_\tau$ nucleon interactions may also lead to secondary taus, pions and muons whose last tracks ( contemporary and parallel muons at a near distance of a few tens of meters) may prove their common tau decay origin.

# 6.  The resonant $\bar{\nu}_e e \to \bar{\nu}_\tau \tau$ and $\bar{\nu}_\tau e \to \tau \nu_e$ events

Finally it is worthful to mention that the important and detectable tau contribution in downward $\bar{\nu}_e e \to W^- \to \bar{\nu}_\tau \tau$ events at energy tuned at the resonant $W^-$ formation mass in $\bar{\nu}_e e$ collisions: $E_\nu^{\rm res} = \frac{M_W^2}{2m_e} = 6.3 \cdot 10^{15}$ eV; at these ranges of energies the muon range is a few kms long while the tau range $R_\tau \sim R_{\tau_o}$ due to an average secondary energy $\langle E_\tau \rangle \simeq \frac{1}{4} E_\nu = 1.4 \cdot 10^{15}$ eV is only $R_\tau \simeq 71$ m. Therefore tau ranges are nearly two orders of magnitude smaller than the those of the muons.

The expected downward muon number of events $N_{eV}(\bar{\nu}_e e \to \bar{\nu}_\mu \mu)$ in this resonant energy range, in $0.2$ km$^3$ detectors, (see Table 7, [2]) was found to be $N_{eV} = 4 \div 7$ a year. We expect a comparable number of reactions $(\bar{\nu}_e e \to \bar{\nu}_\tau \tau)$; however only those events whose originations are confined in $0.2$ km$^3$ volume will be easely recognized as tau ones. Their probability is reduced by a factor related to the corresponding probability to see a confined $\mu$ track inside a km size $\sim \frac{\rm km}{L_\mu} \sim \frac{1}{5}$; therefore roughly an event a year due to reaction $\nu_e e \to \bar{\nu}_\tau \tau$ *might be* noticed by its tau precursor hadronic shower (first "bang", see [3]) and by its probable (64%) secondary contained hadronic cascade (second "bang") as well as by its characteristic range (70m). This (rare) event may occur even in absence of any $\nu_\tau$ cosmic ray and *any* neutrino flavor oscillation. It will be most probable on horizontal tracks where depth size $\simeq$ interaction length; it will not inform us on any important $\nu_\tau$ astrophysics nature, but it must prove, at minimally theoretical assumptions, the same reality of $\nu_\tau$ existence.

Moreover assuming $\nu_\tau$ ultrahigh energy cosmic primary rays at energy range $10^7 > E_{\nu_\tau} > 10^4$ GeV, the same nuclear electroweak $\nu_\tau N$ interactions, (which lead to $\nu_\tau$ (and $\tau$) opacity through the Earth at highest energies), are a source of tau secondaries (even in the energy range where the tau tracks are not longer than the muon ones). Indeed, in the energy range $10^5 \lesssim \left( \frac{E_\nu}{\rm GeV} \right) \lesssim 10^7$ the tau production (by $\nu_e N \to \tau +$ anything) is almost identical to the muon one. The only difference is due to the range length of tau $R_\tau \simeq 50$ m $\left( \frac{E_\tau}{10^6 \, {\rm GeV}} \right)$ to be compared with a few kilometer of a muon $\mu$ radiation range (very sensitive to the exact $E_\mu^{\rm min}$ cut-off).

As before the (detector size/muon range) ratio will offer a first estimate of the ratio of tau/muon contained signals: $R \simeq \frac{1}{5} = 20\%$. (The $\nu_e N$ event rate is *not* suppressed by a much lower ratio $\frac{R_\tau}{R_\mu} \simeq 1\%$). Therefore nearly 20% of the corresponding $\nu_\mu$, $\bar{\nu}_\mu$ events, expected in a km$^3$ telescope, may be associated to tau signals. Only 18% (of these 20% of events) will mask their tau nature by a $\tau \to \mu \nu_\tau \nu_e$ decay, nearly at the same energy direction and therefore hidden in a unique muonic track. Most (82%) of the above events will mark their identity by a 50m $\left( \frac{E_\tau}{10^{16} \, {\rm GeV}} \right)$ tau precursor

track either with a spectacular and characteristic tau-hadronic shower (a jet) ($\sim$64%) or by a short and intense electron shower (whose length, by Landau, Pomeranchuck – Migdal effect [9], is as short as $R_e \cong 4m \left(\frac{E_\nu}{6\cdot 10^{15}\,\text{eV}}\right)^{1/2}$) or, as noted in [3] by their double "bangs". From the arguments above we nearly expect $\sim 20$ atmospheric events a year to be associated with $\tau$ precursor tracks. Moreover, other $\sim 20$ tau events may bring the imprint (and direction) of primary $\nu_\tau$ cosmic rays born in Active Galactic Nuclear (AGN) or mini-galactic jets. These expectations may reach hundred events a year for most optimistic and proliferous spectra of $\nu_\tau$ sources (see [2]). Let us remind that here we neglected all other additional hadronic secondary by ( $\nu_e$, $\nu_\mu$ nucleon elettroweak interactions ) showers that may also decays (by charm or beauty states) in tau leptons.

## Conclusions

A few tau signals a year in a km$^3$ detector *must* occur:

a) at $E_{\nu_\tau} \sim 6 \cdot 10^{15}$ eV energy range, due to $\bar{\nu}_e e \rightarrow \bar{\nu}_\tau \tau$ resonant event, disregarding any primary neutrino $\nu_\tau$ source or even in absence of flavor oscillations.

b) at "low" energies ($E_\nu \simeq 10^5 \div 10^6$ GeV) for any very probable [1] $\nu_\tau$ primary cosmic rays as abundant as $\nu_e$, $\nu_\mu$ ones ($\nu_\tau$ due to charmed hadronic interactions in the source or due to $\nu_e \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow \nu_\tau$ flavor oscillations); we expect from ref [2] and the above approximated arguments, tens of such a $\nu_\tau$ event a year in km$^3$ detectors.

c) At highest energies, a very rare tau signal a year may probe the dominant tau range $10^{12}$ GeV $>$ $E_\tau > 10^8$ GeV. In general, it will cross from size to size the km$^3$ detector, but a few huge hadronic shower in the km$^3$ detectors (comparable to those observed in the rarest atmospheric events ($\sim 3 \cdot 10^{20}$ eV)), may leave a unique imprint: a huge Cerenkov flash (at peak power of Megawatt) due to an initial hadronic shower followed by a collinear (tau) track, whose extension may easily escape the same km$^3$ detector size. However, this rare primary ultrahigh ($E_\nu \gg 10^8$ GeV) neutrino event might be ruled by photopion relics ($\nu_\mu$, $\nu_e$) and therefore it calls for an efficient neutrino flavor oscillation even at widest allowable parameter ranges (see Eq. (1), and [3]) during the neutrino propagation in the Universe.

In conclusion, we believe that in future km$^3$ telescope more surprises may (and must) come from neutrino tau and tau signals: the first direct $\nu_\tau$ experimental evidence, its possible flavor mixing and the first possible spectacular insight at highest energetic ($\gtrsim 10^8 \div 10^{11}$ GeV) neutrino astrophysical frontiers.

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